THE

MATHEMATICAL GAZETTE.

EDITED BY

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WITH THE CO-OPERATION OF

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GENERAL THEORY OF VERNIERS.

I.

1. The practical use and working of a vernier is very simple. It is applicable to all measurements where minute accuracy is required, whether the quantity to be measured is itself a length, as the reading of a barometer, or is determined by the distance passed over by a moving point relatively to a uniformly divided scale straight or curved, as in the angular readings of an astronomical instrument. In its simplest form too, the principle on which it depends is very obvious. Supposing, for instance, we require to read a scale to the one hundredth of an inch. To graduate such a scale directly would require a large number of minute subdivisions, which would be troublesome to make with the requisite accuracy and very confusing to the eye in reading. If, however, the scale be graduated to $\frac{1}{10}$ inch only, and we have an auxiliary scale divided into intervals of $\frac{11}{100}$ inch, sliding alongside of it, which can be adjusted so as to have its upper division at the upper limit B of the quantity to be measured (whose lower limit is at the zero of the principal scale), the fractional amount AB (fig. 1) of the required reading beyond some complete $\frac{1}{10}$ inch, is determined to $\frac{1}{100}$ inch by noting where an exact (or approximate) coincidence between divisions of the two scales occurs. If the rth division from B of the auxiliary scale coincides with the rth from A of the other, the part AB in question $= \dfrac{11r - 10r}{100} = \dfrac{r}{100}$, and the number r gives the number of hundredths required. Thus in the figure the reading is 6·14 ins. Such an auxiliary scale is called a vernier, from its inventor Pierre Vernier (A.D. 1580-1637).

In the above case a vernier division is *larger* than a division of the principal scale by the amount of the minutest subdivision required, but it may equally well be smaller by a like amount. If, for instance, the principal scale be divided to $\frac{1}{20}$ inch, and a vernier interval be $\frac{24}{25}$ of that amount, or $\frac{1}{500}$ inch less, the rth division of the vernier from B will coincide with the $r-1$th from A of the principal scale, when $AB=\dfrac{24r-25(r-1)}{500}=\dfrac{25-r}{500}$ (fig. 2). In other words, the number of 500ths in question is the number of the vernier division which coincides with a division of the other scale, *counting from the 25th upwards* (25 vernier divisions being equivalent to 24 of the principal scale); but the reading is equally simple, only reckoned from the other end of

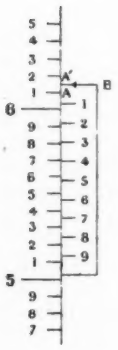

Fig. 1.

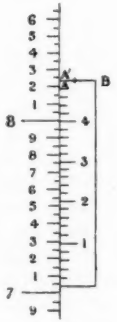

Fig. 2.

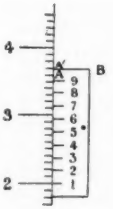

Fig. 3.

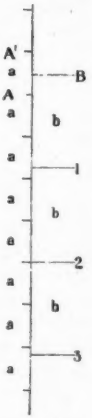

Fig. 4.

the vernier scale to what it was before. For intermediate readings, we should have the rth division (say) of the vernier lying on one side, and the $r+1$th on the other, of two principal divisions, and in this way, with such a vernier as that just described, if finely graduated, the reading could be estimated to one thousandth of an inch, the even thousandths being given by exact coincidences, and the odd thousandths by symmetrically intermediate positions. (The reading shown in the figure is 8·236 ins., ·036 being the fraction corresponding to a coincidence with the 18th vernier division from its lower end.) The two cases above illustrate the simplest kinds of backward-reading and forward-reading verniers respectively, but we may, if we please, have a vernier interval a little greater or less than not one, but two or more, principal intervals. Suppose a division of

the principal scale $= \frac{1}{10}$ cm. and of the vernier $\frac{19}{100}$ cm. (fig. 3). Then $AB = \dfrac{19r - 10(2r-1)}{100}$ cm. $= \dfrac{10-r}{100}$ cm. when the rth division from B in the vernier coincides with the $\overline{2r-1}$th from A in the principal scale; and similarly in other cases. (In the figure we read 3·71 cm., the vernier reading forwards, as in fig. 2.)

2. Such are the practical results with which we have to deal. They are obviously cases of the following general question. Suppose two scales uniformly divided into intervals a, b etc., placed one against the other, so that the initial division B of the second lies between two consecutive divisions A, A' of the first, while the qth division from A in the one coincides with the pth from B in the other (fig. 4): how is the magnitude of the interval AB connected with the numbers p and q? We have

$$AB = qa - pb, \text{ or } AB = pb - qa,$$

according as A is the division which precedes or which follows B (in the direction in which the vernier extends from B). The figure illustrates the latter case, but the former is exactly similar. Supposing a, b to be commensurable, and that c is their G.C.M.; so that $a = mc$, $b = nc$, where m, n are integers prime to each other. Then $AB = rc$, where $pn - qm = r$ $(r < m)$. The relation in question is thus defined by the fundamental indeterminate equation of the first degree. Considering the relation between r and p only, we have $pn - r$ divisible by m, or $pn \equiv r \pmod{m}$; and as p takes the successive values from 1 to $m-1$, r will take the same series of values *in some order*, so that a value of AB equal to any integral multiple of c will give a coincidence of some definite division of the second scale (between B and the mth from B), with some division of the first. If then we require such values to include all multiples of a given part c of $a = \dfrac{a}{m}$, we may take for b any multiple $\dfrac{na}{m}$ of c, where n is prime to m: the case $n = 1$ reduces merely to simple subdivision of a. But in general the connection between the values of p and r which thus definitely correspond to each other will not be obvious at a glance. If, in fig. 4, $b = \frac{3}{16}a$, coincidences of the 1st, 2nd . . . 15th b-divisions will correspond to values of $AB = 3, 6, 9, 12, 15, 2, 5, 8, 11, 14, 1, 4, 7, 10, 13$ sixteenths of a respectively. Thus the value of AB in the figure would be $\frac{6}{16}a$. For the indications to be perfectly obvious we must take $n = km \pm 1$, i.e. $b = \dfrac{km \pm 1}{m}\, a$. Then in the one case $p = r$, in the other $p = m - r$. If, *e.g.*, in fig. 4, $b = \frac{1}{5}a$, the coincidences of the 1st, 2nd, 3rd, 4th of the b-divisions correspond to readings $\frac{1}{5}$, $\frac{2}{5}$, $\frac{3}{5}$, $\frac{4}{5}$ of a; AB in the figure being $= \frac{2}{5}a$. And all the practical examples given above

come under the same head. In the first case $m=10$, $n=m+1$; in the second $m=25$, $n=m-1$; in the third $m=10$, $n=2m-1$; while in that just noticed $m=5$, $n=2m+1$. Whatever m may be, $n=km\pm1$ (with any integral value of k) secures that the vernier reads regularly, either backwards or forwards.

We may observe that as, in the general case, for p and r, so between q and r we have a relation determined by $qm\equiv-r$ (mod. n), but the values of r from 1 to $m-1$ will not correspond in the same unique way with the successive values of q as with those of p. The complete series of values of q from 1 to $n-1$ will always be in excess or defect of what are required to define the subdivisions of a into m equal parts. If $m>n$ there will be different values of r corresponding to the same values of q: if $m<n$, there will be values of q, to which no values of $r, <m$, correspond. In the one case, $a>b$, there will be divisions of the primary scale which are in coincidence with divisions of the other for more than one position of B between A, A': in the other, $a<b$, there will be divisions of the primary scale which will not come into coincidence with divisions of the other for any position of B between A, A'. Thus neither in fig. 3, nor in fig. 4, would the third division from A be involved in a coincidence for any position of B within such a range. For this general reason, as well as for others of practical convenience, we reckon by divisions of the vernier, not by those of the primary scale, in estimating the fractional part of a in question, as in the practical examples considered above. In length the vernier must $=mb=na$. It is unnecessary to compare the scales beyond this range, as all the circumstances will repeat themselves after such an interval.

For more general results founded on the relations between two contiguous scales, which illustrate in an interesting way the fundamental theorems of continued fractions, we adopt a slightly different point of view.

PERCY J. HEAWOOD.

DURHAM, *October*, 1902.

(*To be continued.*)

MATHEMATICAL NOTES.

109. [K. 6. a.] *Trilinear Notes.*

(1) If $\alpha_1, \beta_1, \gamma_1$ and $\alpha_2, \beta_2, \gamma_2$ are the *trilinear* coordinates of any two points on the straight line $l\alpha+m\beta+n\gamma=0$, then

$$l(\alpha_2-\alpha_1)+m(\beta_2-\beta_1)+n(\gamma_2-\gamma_1)=0,$$

$$a(\alpha_2-\alpha_1)+b(\beta_2-\beta_1)+c(\gamma_2-\gamma_1)=0,$$

then

$$\frac{\alpha_2-\alpha_1}{cm-bn}=\frac{\beta_2-\beta_1}{an-cl}=\frac{\gamma_2-\gamma_1}{bl-am};$$

now $D^2\sin^2C=(a_2-a_1)^2+(\beta_2-\beta_1)^2+2(a_2-a_1)(\beta_2-\beta_1)\cos C$,

where D is the distance between the points;

$\therefore$ each of the above fractions

$$=\sqrt{\left\{\frac{D^2\sin^2C}{(cm-bn)^2+(an-cl)^2+2(cm-bn)(an-cl)\cos C}\right\}},$$

$$\left.\begin{aligned}\text{denominator}&=c^2m^2-2bcmn+b^2n^2+a^2n^2-2acnl+c^2l^2\\&\quad+2\cos C(acmn-abn^2-c^2lm+bcnl)\end{aligned}\right\}$$

$$=c^2\{\Sigma l^2-2\Sigma mn\cos A\}$$

$$=c^2T^2;$$

$\therefore$ each of the above fractions is $=(D\sin C)/(cT)=D/(2RT)$.

Thus the sines of the angles made by the straight line $la+m\beta+n\gamma=0$ with BC, CA, AB respectively are

$$(m\sin C-n\sin B)/T,\quad(n\sin A-l\sin C)/T,\quad(l\sin B-m\sin A)/T,$$

the cosines are therefore $(l-m\cos C-n\cos B)/T$, etc., for

$$(l-m\cos C-n\cos B)+(m\sin C-n\sin B)^2=T^2.$$

(2) If θ is the angle between the two straight lines $\left.\begin{aligned}l_1a+m_1\beta+n_1\gamma=0\\l_2a+m_2\beta+n_2\gamma=0\end{aligned}\right\}$

(θ = difference of angles made by straight lines with BC),

$$\begin{aligned}T_1T_2\sin\theta&=(m_1\sin C-n_1\sin B)(l_2-m_2\cos C-n_2\cos B)\\&\quad-(m_2\sin C-n_2\sin B)(l_1-m_1\cos C-n_1\cos B)\\&=(m_2n_1-m_1n_2)\sin A+\ldots+\ldots,\end{aligned}$$

$$\begin{aligned}T_1T_2\cos\theta&=(l_1-m_1\cos C-n_1\cos B)(l_2-m_2\cos C-n_2\cos B)\\&\quad+(m_1\sin C-n_1\sin B)(m_2\sin C-n_2\sin B)\\&=l_1l_2+m_1m_2+n_1n_2-(m_1n_2+m_2n_1)\cos A\ldots.\end{aligned}$$

(3) If P is (f, g, h) and PN perpendicular on $la+m\beta+n\gamma=0$ (making angles ϕ_1, ϕ_2, ϕ_3 with BC, CA, AB), and coordinates of N are a_1, β_1, γ_1,

$$PN=(f-a_1)\cos\phi_1=(g-\beta_1)\cos\phi_2=(h-\gamma_1)\cos\phi_3;$$

$$\therefore\quad PN=(lf+mg+nh)/(l\cos\phi_1+m\cos\phi_2+n\cos\phi_3),$$

since $$la_1+m\beta_1+n\gamma_1=0,$$

$$\text{denominator}=\frac{l(l-m\cos C-n\cos B)}{T}+\ldots+\ldots=\frac{T^2}{T}=T;$$

$$\therefore\quad\text{perpendicular}=(lf+mg+nh)/T.$$

$$\text{Equation to } PN \text{ is } \begin{vmatrix}a,&f,&a_1\\\beta,&g,&\beta_1\\\gamma,&h,&\gamma_1\end{vmatrix}=\begin{vmatrix}a,&f,&f+PN\cos\phi_1\\\beta,&g,&g+PN\cos\phi_2\\\gamma,&h,&h+PN\cos\phi_3\end{vmatrix}$$

$$=\begin{vmatrix}a,&f,&\cos\phi_1\\\beta,&g,&\cos\phi_2\\\gamma,&h,&\cos\phi_3\end{vmatrix}=\begin{vmatrix}a,&f,&l-m\cos C-n\cos B\\\beta,&g,&m-n\cos A-l\cos C\\\gamma,&h,&n-l\cos B-m\cos A\end{vmatrix}=0.$$

R. F. Davis.

110. [L¹. 3. a.] *Method of reducing Central Conics.*

The coordinate axes being inclined at any angle ω, let the ellipse

$$ax^2+2hxy+by^2+2gx+2fy+c=0$$

be identical with

$$ax^2+2hxy+by^2+2\sqrt{ab-h^2}(xy_2-x_2y)+c=0,$$

the point (x_1, y_1) being the centre, so that

$$\frac{-x_1}{hx_2+by_2}=\frac{y_1}{hy_2+ax_2}=\frac{1}{\sqrt{ab-h^2}},$$

and $$gx_1+fy_1=\sqrt{ab-h^2}(x_1y_2-x_2y_1)=-(ax_2^2+2hx_2y_2+by_2^2)=\frac{[\nabla]_{c=0}}{ab-h^2},$$

since the discriminant

$$\nabla=(ab-h^2)\{c-(ax_2^2+2hx_2y_2+by_2^2)\}.$$

Hence also $$[\nabla]_{c=-(gx+fy)}=-(ab-h^2)\{(gx+fy)-(gx_1+fy_1)\}$$
$$=-(ab-h^2)^{\frac{3}{2}}\{(xy_2-x_2y)-(x_1y_2-x_2y_1)\}.$$

But $(ab-h^2)\{(xy_2-x_2y)^2+(xy_1-x_1y)^2\}=(ax^2+2hxy+by^2)(ax_2^2+2hx_2y_2+by_2^2)$,

and $$-2(ab-h^2)(xy_2-x_2y)(x_1y_2-x_2y_1)=(2gx+2fy)(ax_2^2+2hx_2y_2+by_2^2).$$

Also if $\nabla=\mu[\nabla]_{c=0}$, so that $c=(1-\mu)(ax_2^2+2hx_2y_2+by_2^2)$,

then $$(ab-h^2)(1-\mu)(x_1y_2-x_2y_1)^2=c(ax_2^2+2hx_2y_2+by_2^2).$$

Thus the equation of the ellipse becomes

$$\{(xy_2-x_2y)-(x_1y_2-x_2y_1)\}^2+(xy_1-xy_1)^2=\mu(x_1y_2-x_2y_1)^2,$$

or $$[\nabla^2]_{c=-(gx+fy)}+(ab-h^2)^3(xy_1-x_1y)^2=\nabla[\nabla]_{c=0},$$

and $$\frac{(xy_2-x_2y)-(x_1y_2-x_2y_1)}{\cos\theta}=\frac{xy_1-x_1y}{\sin\theta}=\sqrt{\mu}(x_1y_2-x_2y_1)$$

is a point on the curve whose coordinates are

$$x=x_1+\sqrt{\mu}(x_1\cos\theta-x_2\sin\theta)$$

and $$y=y_1+\sqrt{\mu}(y_1\cos\theta-y_2\sin\theta).$$

Let r_1, r_2, and r_3 be the distances of the points (x_1, y_1), (x_2, y_2), and (x_1+x_2, y_1+y_2) from the origin.

The area of the parallelogram which has these four points at its angles is $\pm(x_1y_2-x_2y_1)\sin\omega$, and it is contained by conjugate semi-diameters of

$$ax^2+2hxy+by^2+2gx+2fy=0,$$

and the tangents at the extremities (0, 0) and (x_1+x_2, y_1+y_2).

The semi-diameters are inclined at an angle $\cos^{-1}\left(\frac{r_1^2+r_2^2-r_3^2}{2r_1r_2}\right)$.

Let $(xy_1-x_1y)=0$ and its conjugate be taken for axes of ξ and η.

Then $\xi=r_1$ and $\eta=r_2$ may be taken as the new coordinates of the point (x_2, y_2); and we shall have

$$\frac{\xi}{r_1}=\frac{(xy_2-x_2y)-(x_1y_2-x_2y_1)}{-(x_1y_2-x_2y_1)},\quad \frac{\eta}{r_2}=\frac{xy_1-x_1y}{-(x_1y_2-x_2y_1)},$$

and the equation of the ellipse will be

$$\frac{\xi^2}{r_1^2}+\frac{\eta^2}{r_2^2}=\mu,$$

whence we see that if $\frac{X^2}{\rho_1^2}+\frac{Y^2}{\rho_2^2}=\mu$ is the equation in reference to the principal axes, $2\rho_1\sqrt{\mu}$ being the major axis, ρ_1 and ρ_2 will be the roots of the equation

$$\rho^4-(r_1^2+r_2^2)\rho^2+(x_1y_2-x_2y_1)^2(\sin\omega)^2=0.$$

Also the auxiliary circle will be

$$\xi^2+\frac{r_1^2+r_2^2-r_3^2}{r_1r_2}\xi\eta+\eta^2=\mu\rho_1^2.$$

Consequently

$$\frac{\xi^2}{r_1^2}(\rho_1^2-r_1^2)-\frac{\xi\eta}{r_1r_2}(r_1^2+r_2^2-r_3^2)+\frac{\eta^2}{r_2^2}(\rho_1^2-r_2^2)=0$$

represents two lines which are coincident with the major axis, whose equation therefore is

$$2\frac{\xi}{r_1}(\rho_1^2-r_1^2)-\frac{\eta}{r_2}(r_1^2+r_2^2-r_3^2)=0,$$

or $$2(\rho_1^2-r_1^2)[\nabla]_{c=-(gx+fy)}+(ab-h^2)^{\frac{1}{2}}(r_1^2+r_2^2-r_3^2)(xy_1-x_1y)=0.$$

Similarly the minor axis is

$$2(r_1^2-\rho_2^2)[\nabla]_{c=-(gx+fy)}-(ab-h^2)^{\frac{1}{2}}(r_1^2+r_2^2-r_3^2)(xy_1-x_1y)=0.$$

Similar results may be obtained for the hyperbola, and its asymptotes will be

$$\begin{vmatrix} x, & y, & 1 \\ -f, & g, & \pm\sqrt{h^2-ab} \\ bg-fh, & af-gh, & h^2-ab \end{vmatrix}=0,$$

except in the case when one asymptote passes through the origin.

115. [J. 1. a.] *The number of ways of writing "Tot tibi sunt dotes, Virgo, quot sidera coelo," without disobeying the laws of metre, caesura excepted* (*v. No.* 35, *p.* 216, *Netto's "Lehrbuch der Combinatorik"*).

Let α denote a monosyllable, β a dissyllable, and γ the alternative of *sidera* or *tibi*.

Then we have $aa\alpha\beta\beta\beta\gamma\gamma$.

I. Let the verse end with $\gamma\beta$. The β can be chosen in three ways; the γ in two ways. It remains to arrange the remaining words $aa\alpha\beta\beta\gamma$.

These can be arranged in $\lfloor\overline{6}$ orders, of which exactly half will be valid. For if γ=*sidera* those arrangements will be valid which have an even number of α's somewhere before the γ; and if γ=*tibi* those will be valid which have an odd number of α's somewhere before the γ.

Hence choice $=3\times2\times\lfloor\overline{6}\div2=2160$.

II. Let the verse end with γaa. The aa can be chosen in six ways, the γ in two ways. It remains to arrange the remaining five words. And, precisely as before, half of the $\lfloor\overline{5}$ arrangements will be valid.

Hence choice $=6\times2\times\lfloor\overline{5}\div2=720$.

Therefore the total number of arrangements is $2160+720=2880$.

If it were required to have caesura in the third foot the number of arrangements would be reduced to 1620. W. Allen Whitworth.

The words in the line give the syllabic arrangement

— ⏑ ⏑	— ⏑ ⏑	— ⏑ ⏑	— ⏑ ⏑	— ⏑ ⏑	— ⏓
or	or	or	or		
— —	— —	— —	— —		

I. Line ends in *sidera* followed by a dissyllable (not *tibi*). The dissyllable may be chosen in three ways: for each of the $\lfloor\overline{6}$ permutations of the remaining six words we have to reject those in which *tibi* is first or third in order of the four words *tot tibi sunt quot*, *i.e.* half the whole number.

I. therefore gives $\frac{3\lfloor\overline{6}}{2}$ arrangements.

II. Line ends in *sidera* followed by two monosyllables. The sixth foot can be formed in six ways: for each case, from the whole number of permu-

tations of the remaining five words, we have to reject all in which *tibi* comes earlier than the remaining monosyllable—half of the whole number.

$\therefore$ II. gives $\frac{6\lfloor 5}{2}$ arrangements.

III. Line ends in *tibi* followed by dissyllable: from the possible permutations of the words we have to reject those in which *sidera* comes second or fourth of the words *tot sunt quot sidera*.

$\therefore$ III., like I., gives $\frac{3\lfloor 6}{2}$ arrangements.

IV. Line ends in *tibi* followed by two monosyllables: from possible permutations reject those in which a monosyllable comes earlier than *sidera*.

$\therefore$ IV. gives $\frac{6\lfloor 5}{2}$ arrangements.

$\therefore$ Whole number of arrangements is $2\left(\frac{3\lfloor 6}{2}+\frac{\lfloor 6}{2}\right)=4\lfloor 6=2880$.

W. E. HARTLEY.

116. [X. 4. b. α.] *Note on the Graphical Solution of Quadratics.*

The general quadratic, $ax^2+bx=c$, is satisfied by the values of x which are common to the equations $y=x^2$, $ay+bx=c$.

The first of these represents a fixed parabola which is easily graphed for permanent use, and the second represents a variable straight line which is easily graphed for particular values of a, b, c. Thus the only requisites for approximating to the roots of a quadratic are the possession of a good printed, or accurately drawn, graph of the above parabola on squared paper and a little ingenuity in the manipulation of the straight line.

Two points only on the line $ay+bx=c$ are wanted, and these are generally to be obtained by selecting simple integral values for x and y.

E.g., convenient points on $3y+5x=4$ would be $\left.\begin{matrix} x=2 \\ y=-2 \end{matrix}\right\}$, $\left.\begin{matrix} x=-1 \\ y=3 \end{matrix}\right\}$.

In practice it will be found convenient not to draw the line, but to place the straight edge of a sheet of paper on the points selected (which are usually two of the corners already marked on the squared paper), and prick on the parabola the points where it is cut by the straight edge.

In this way the original graph can, with care, be used for a considerable time. Of course any parabola would do, but $y=x^2$ has the additional merit of providing a rough table of squares and square roots.

In a similar way the graph of $y=x^3$ can be used to get solutions to any cubic equation if the cubic be first thrown into the form $x^3+ax=b$.

A. E. WYNNE.

[The reader may remember the semi-graphic solution of a quartic equation by Prof. G. B. Mathews in *Nature*, Nov. 16, 1899. In Mr. Hall's supplementary chapter on "Graphical Algebra" to Hall and Knight's elementary treatise (6d., Macmillan), we find: Plot the graphs of $y=x^3-3x$. Find the roots of $x^3-3x=0$, with several similar questions, pp. 22, 23. W. J. G.]

REVIEWS.

Higher Mathematics for Students of Chemistry and Physics. By J. W. MELLOR, D.Sc. (Longmans.)

This is a most remarkable book. Almost the whole range of the mathematics required in connection with physical chemistry is to be found in it, admirably arranged and illustrated by an extraordinary variety of formulae and examples,

which are used to give point and interest to the mathematical work. Felicitous and stimulating quotations from great authorities are to be found throughout the book, and many references are given to standard books and papers to be consulted by students who desire to go more deeply into some particular branch of the subject. Lists of formulae of various kinds are given at the end of the book, and also sets of tables of the numerical values of some of the more important functions. An excellent index is appended.

But the utility of the book is greatly marred, and in places almost destroyed by the most extraordinary carelessness in the mathematical explanations, and in the formulae, so serious as to make the book unreadable by a beginner.

Lists of errata might do some good, but the blemishes could be completely removed only by means of a new edition. There are also one or two persistent mis-spellings, which are somewhat irritating, such as 'trignometry' and 'auxillary.'

A few of the mathematical errors are indicated below.

p. 29 (u is a function of y, and y is a function of x). "Let u and y receive small increments so that $y_1 = y + h$, and $u_1 = u + h$, and $x_1 = x + h$."

p. 55 (Speaking of the expansion, under heat, of a cuboid whose sides are of lengths x, y, z). The author gives $dx/d\theta$, $dy/d\theta$, $dz/d\theta$ as the coefficients of linear expansion instead of $\frac{1}{x} \cdot \frac{dx}{d\theta}$, etc., and says that the coefficient of cubical expansion is obtained by putting $x = y = z = 1$.

p. 195, last two lines. This explanation of double integration is entirely erroneous and misleading. On the next page, the explanation of triple integration is better, except that the word 'area' is used instead of 'volume.'

pp. 221, 222. The author shows (p. 221) how to find the rate of diffusion of a gas under certain specified conditions, but in a cumbrous manner and incompletely, and then on the next page gives a numerical example in which he makes no use whatever of his previous work, but takes *quantity diffused in a given time* instead of *rate of diffusion*.

p. 223. In establishing an empirical formula of the kind, $y = A + Bx + Cx^2$, from numerical data, the author takes two degrees of approximation, viz.,

$$(1)\ y = A + Bx, \quad \text{and} \quad (2)\ y = A + Bx + Cx^2,$$

using the *same* values of A and B in each case, so that every value of y calculated by (1) comes out too small! Surely this would not help a student to understand how these formulae should be arrived at.

p. 240. The author appears to say that if h and k be taken sufficiently small, an expression of the form $ah^2 + 2bhk + ck^2$ will always have the same sign.

p. 244. The author asks the student to prove that

$$\operatorname{Lt}_{n=-1} \frac{x^{n+1}}{n+1} = \log x.$$

p. 245. To evaluate $x \log x$ when $x = 0$, the author says: "Transpose the infinite term to the denominator and differentiate. $x/\log x$ becomes on differentiation x^2; $\therefore \operatorname{Lt}_{x=0} x \log x = 0$."

p. 247. In Stirling's formula, x is used for the *excess* of x above x_0. The same inaccuracy occurs on p. 251 in Newton's formula.

p. 252. The meanings of the symbols used are confused and mixed, and then, to crown all, the equation (4), which he gives as an illustration of Lagrange's method of interpolation, is something entirely different; most ingenious and interesting, but, as explained, absolutely misleading.

p. 272. The author's language is not clear, but he appears to state that if A is the vertex, and P any other point on the hyperbola $x^2 - y^2 = a^2$, then, if $x = a \cosh u$, $y = a \sinh u$, the value of u is the ratio of the arc AP to the radius vector OP.

There are other errors or misleading statements of the same kind throughout the book, and besides these there are innumerable minor errors, *i.e.* such as could be corrected by a table of errata, but which, till corrected, will be a source of constant perplexity and irritation to the reader.

In spite of these errors, however, the book is so well arranged, so stimulating, so full of ideas, as to make it fascinating reading, and it is to be hoped that the author will do what he can to eradicate these defects. ALFRED LODGE.

Applied Mechanics for Beginners. By J. DUNCAN. (Macmillan.)

A pleasant and well-written introduction to the subject, with excellent diagrams.

The author expects the students to be taken through an experimental course, such as is sketched out in the book, or to be practically engaged in works.

No mathematical knowledge is assumed beyond elementary algebra and the ability to use logarithms. The student should also know something of practical geometry and machine and building construction.

The book is intended to cover the requirements for the elementary examination in applied mechanics of the Board of Education.

At the end will be found tables of four-figure logarithms and a useful index.

A. LODGE.

An Elementary Treatise on Kinematics and Dynamics. By J. G. MACGREGOR. Pp. xvi., 525. 10s. 6d. 2nd edition. 1902. (Macmillan.)

An Elementary Treatise on Theoretical Mechanics. By A. ZIWET Pp viii., 181; viii., 184; viii., 236. 21s. net. 2nd edition. 1902. (Macmillan.)

The Science of Mechanics. By E. MACH. (Translated from the German by T. J. M'CORMACK.) Pp. xx., 605. 9s. 6d. 2nd edition. (Open Court, Chicago; and Kegan Paul.)

Dynamics of Rotation. By M. WORTHINGTON. Pp. viii., 164. 4s. 6d. 1902. (Longmans.)

It is not necessary to do much more than to call attention to the issue of further editions of the above works. For the convenience of those who possess the first edition of Mr. Macgregor's treatise we may refer them to the improvements in sections 223, 289, 290, 298, 301, and also to additions, corrections, etc., on pp. 188, 190, 192, 193, 198, 207, 209, 259, 305, 319, 464, 492, 493, and 592. We may have overlooked a few verbal alterations, but we think that all the important renovations will be included in this list.

Professor Ziwet's book consists of the three original volumes bound in one. It is a good book for the engineer, and is conceived on purely orthodox lines.

Mach's *Die Mechanik in ihrer Entwickelung historisch-kritisch dargestellt* needs no bush to declare its superlative merit. No one can question the dictum of Professor Love in the preface to his introductory treatise on the principles of dynamics: "The works which have been most useful to me in matters of principle are . . . Mach's *Science of Mechanics*. The last should be in the hands of all students who desire to follow the history of dynamical ideas." We may add that the additions to the new issue are considerable in amount, and interesting as illustrating the author's attitude to the Hertzian standpoint.

Mr. Worthington has made some important changes in the fourth edition of his little text-book on rotation. A section is inserted on the equal and similar centrifugal couples of equimomental bodies similarly rotating—the results being used in the Appendix in connection with the top and gyroscope. The author lays special stress on his favourite "Inertia skeleton," replacing the momental ellipsoid.

Examples in Algebra. By C. O. TUCKEY. Pp. viii., 178. 3s. 1902. (Bell & Sons.)

Easy Mathematical Problem Papers. By C. DAVISON. Pp. vi., 120. 2s. 6d. (Blackie.)

A New Sequel to Euclid. By W. J. DILWORTH. Pp. vi., 196. 2s. 6d. N.D. (Blackie.)

Mr. Tuckey leaps in to fill one of the gaps formed by the new order of things of which the Mathematical Association may fairly say "quorum pars magna fui." Graphs, checks on accuracy, oral questions, algebra applied to elementary physics, to geometry, to mensuration, all contribute to make this a most useful aid to the teacher. On page 92, the numbers 1-7 should be placed in brackets, and the same slip is to be found on page 91, Ex. 72, Nos. 1-5 and 6-10. The ground covered extends to the Binomial Theorem. But what a criticism it is on our national conservatism that every French schoolboy has been doing his graphs and all the rest of it at this stage of his work for at least the last twenty years! We would venture to suggest that maxima and minima should not be omitted from a future edition.

Mr. Davison covers the same ground in Algebra as Mr. Tuckey. In Euclid he takes Bks. I.-IV., VI., XI., and Trigonometry up to Solution of Triangles. They are good of their kind, quite of the ancient type save that there is more Arithmology than usual. No graphs seem to be asked for.

Mr. Dilworth's *New Sequel to Euclid* is not dated, but would appear to be a new issue without alterations. It is an excellent introduction to the type of questions in geometry identified with Irish examination papers. The author's ideas on the teaching of geometry are not on all fours with progressive British opinion. "An experience of sixteen years as a teacher has convinced me that until the first three books of Euclid have been mastered by the pupil, it is unwise to bring before him any but the simplest corollaries to the propositions of Euclid." Accordingly, after mastering the first three books of Euclid, the student is invited to prove that "the bisector of the vertical angle of an isosceles triangle bisects the base at right angles"! In spite of the unfortunate experiences of Mr. Dilworth as a teacher, his book of riders is skilfully constructed, and should be found very useful for class-work in the hands of a judicious teacher.

PROBLEMS.

450. [K. 20. e.] In any triangle, r_1, r_2, r_3 being the ex-radii, and s the semi-perimeter, show that

$$\frac{r_2+r_3}{(s-a)\sin A}=\frac{r_3+r_1}{(s-b)\sin B}=\frac{r_1+r_2}{(s-c)\sin C}$$

and give a symmetrical expression for the common value. E. N. Barisien.

451. [J. 1. a. α.] If $f(n)$ be the number of permutations of n letters altogether, on condition that no letter is moved more than one place from its original position in a given order, then

$$L_{n=\infty}\frac{f(n)}{f(n+1)}=2\sin 18^\circ.$$

E. M. Langley.

452. [L¹. 16. a.] BB' is the minor axis of an ellipse, whose centre is O, and C is the mid-point of OB': find the locus of the orthocentre of the triangle PBC, P being a variable point on the ellipse. V. Retali.

453. [L¹. 17. e.] Two conics have a common focus, and their directrices are at right angles: what is the condition that their common chord may pass through the common focus? A. F. Van der Heyden (Durham, 1902).

454. [K. 12. b. α.] Three circles touch one another at A, B, C: show that the radii of the circles which touch all three are $R/(\cot\omega\pm 2)$; R being the circum-radius of ABC. C. E. Youngman.

455. [R. 8. 1.] A circular hoop of mass M rests with its plane vertical in a horizontal plane; two small smooth rings, each of mass m, start together from rest at the highest point of the hoop and slide down it: show that the hoop will leave the plane when the radius vector to each ring makes with the vertical an angle

$$\cos^{-1}\left(\frac{1+\sqrt{1-3M/2m}}{3}\right).$$

If $M=0$ investigate the subsequent motion. (Melbourne, 1896.)

456. [A. 1. c.] In $(\sum_1^m x_m)^n$, where n is a positive integer and $n=mk+r$, how many terms have equal coefficients greater than the other terms of the expansion? (Jesus (C.), 1897.)

457. [L¹. 3. d.] A line through the focus perpendicular to an asymptote cuts the hyperbola again in P, Q. Find by polars the value of $SP.SQ$. (Jesus (C.), 1897.)

458. [R. 1. a.] Two equal particles are joined by a light inextensible string stretched perpendicular to the edge of a smooth horizontal table. One particle is gently pushed over the edge. At what depth below the surface of the table will the whole string be horizontal? (St. John's (C.), 1897.)

459. [L¹. 17. e.] Find the locus of the centres of conics having double contact with each of two homothetic conics. (Trin. C., 1897.)

SOLUTIONS.

THE following problems are still unsolved: Vol. I., 171, 275, 279, 283, 326, 336-8, 370, 373-4, 376-9, 381-2, 389, 395, 397; Vol. II., 404, 422-3, 425-7, 430-1, 436-9, 440, 443, 445, 446-8. Solutions to these, or other questions to which solutions have not been published, should be sent as early as possible.

The number should precede the solution, but the question need not be re-written. Figures should be **very carefully drawn to a small scale on a separate sheet.**

393. [M₁. 2. a.] *Salmon, Higher Plane Curves, p.* 148, *footnote:* "*It is easy to see that we may have nine real points lying by threes on ten straight lines but not in a greater number of lines.*"

Given four points A, B, C, D, not all in a straight line, give a geometrical construction for the remaining five points to complete a set satisfying the above conditions. C. S. JACKSON.

Solution by J. BLAIKIE.

Let A, B, C be three points in a straight line, D a point not in the line. Take A', B', C', so that AA', BB', CC' are bisected at D, and E, E' the intersections of AB' with BC' and $A'B$ with $B'C$. Then $BEB'E'$ is a parallelogram and E, D, E' are collinear.

This is, of course, a particular case; the more general case in which A', B' are any points on AD and BD may be derived from it by projection.

Solution by PROPOSER.

Let A, B, C, D be the given points, F the intersection of BC with AD, H, G the intersections of BD and AC respectively with the diagonal through F of the quadrilateral determined by A, B, C, D, and I, K the intersections of AB with HC and BG with IF. Then D, C, K are collinear.

Prove by projecting A, B, G, F into a parallelogram.

[If two triangles $A_0A_1A_2$, $B_0B_1B_2$ be such that A_0B_0, A_1B_1, A_2B_2 concur in C_0 and A_0B_1, A_1B_2, A_2B_0 in C_1, then applications of Menelaus's theorem to the sides of $B_0B_1B_2$ and the two known axes of perspective will shew that there is a third axis of perspective, and A_0B_2, A_1B_0, A_2B_1 concur in a third point C_2. The relation between the three triads of points is symmetrical, and given one triad and two points of another, the remaining points are determined by the intersections of straight lines.

The most general solution to the problem proposed would therefore be given by such a set of points in which the first known triad are collinear, *e.g.*, if no three of the four given points be collinear we may take for the fifth any point on the line joining two of the given ones.]

398. [K. 10. e.] *The square of the tangent from the origin to the circle circumscribing the triangle formed by the lines*

$$lx+my=1, \quad mx+ny=1, \quad nx+ly=1$$

is

$$\Sigma(m^2+n^2)(mn-l^2)/\Sigma mn(m^2+n^2)(mn-l^2). \qquad \text{(C.)}$$

Solution by D. LINTSOF.

The equation to the circumcircle of ABC is

$$\frac{\sin A}{\alpha}+\frac{\sin B}{\beta}+\frac{\sin C}{\gamma}=0,$$

In our case $$a=\frac{mx+ny-1}{\sqrt{m^2+n^2}}, \text{ etc.},$$

$$\sin A=\frac{mn-l^2}{\sqrt{l^2+m^2}\sqrt{l^2+n^2}}, \text{ etc.}$$

(We may see that proper signs have been taken by noticing that the coefficients of x^2 and y^2 are equal.) The square of the tangent from the origin is the quotient of the constant term by the coefficient of x^2; and is

$$=\frac{\sin A\sqrt{m^2+n^2}+\sin B\sqrt{n^2+l^2}+\sin C\sqrt{l^2+m^2}}{nl\sin A\sqrt{m^2+n^2}+lm\sin B\sqrt{n^2+l^2}+mn\sin C\sqrt{l^2+m^2}}$$

$$=\frac{\Sigma(m^2+n^2)(mn-l^2)}{\Pi(mn-l^2)},$$

the value given in the enunciation being incorrect.

We may notice that if Δ be the area, R the circum-radius of the triangle,

$$2\Delta\times\Pi(mn-l^2)=\begin{vmatrix} l, & m, & 1 \\ m, & n, & 1 \\ n, & l, & 1 \end{vmatrix}^2$$

$$R\times\Pi(mn-l^2)=\begin{vmatrix} l, & m, & 1 \\ m, & n, & 1 \\ n, & l, & 1 \end{vmatrix}\times\Pi\sqrt{m^2+n^2}.$$

If (X, Y) be any point on the circumcircle of the triangle formed by

$$lx+my=1, \quad l'x+m'y=1, \quad l''x+m''y=1,$$

and $$px+qy+r=0,$$

its Simson line with respect to the triangle, then since

$$lx+my=1, \quad mx-ly=mX-lY, \quad px+qy+r=0$$

are concurrent,

$$p(l+m^2X-lmy)+q(m-lmX+l^2Y)+r(l^2+m^2)=0.$$

Hence the equation to the circumcircle of the triangle indicated is given by the determinant

$$[l+m^2X-lmY\,.\,m'-l'm'X+l'^2Y\,.\,l''^2+m''^2]=0,$$

whence the result may be obtained for the general triangle.

Solution by R. TUCKER.

The circumcircle of

$$a_1x+b_1y+c_1=0, \text{ etc.},$$

is $$(x^2+y^2)\Pi(a_2b_3-a_3b_2)+x\Sigma c_1(a_2b_3-a_3b_2)(a_1a_2a_3+\Sigma a_1b_2b_3-2a_1b_2b_3)$$
$$+\ldots-\Sigma c_2c_3(a_2b_3-a_3b_2)(a_1^2+b_1^2)=0.$$

Hence in the case given the square of the tangent from the origin to the circle is

$$\Sigma(m^2+n^2)(mn-l^2)/\Pi(mn-l^2).$$

402. [B. 1. a.] *Prove that the determinant*

$$\begin{vmatrix} a_0, & a_1x, & a_2x^2, & \ldots & a_nx^n \\ a_nx^n, & a_0, & a_1x, & \ldots & a_{n-1}x^{n-1} \\ a_{n-1}x^{n-1}, & a_nx^n, & a_0, & \ldots & a_{n-2}x^{n-2} \\ \ldots & \ldots & \ldots & \ldots & \ldots \\ a_1x, & a_2x^2, & a_3x^3, & \ldots & a_0 \end{vmatrix}$$

is a polynomial of order n in x^{n+1}. F. S. MACAULAY.

Solution by PROPOSER.

Multiply the given determinant Δ by

$$\Delta' = \begin{vmatrix} 1, & 1, & 1, & \dots & 1 \\ 1, & \omega, & \omega^2, & \dots & \omega^n \\ 1, & \omega^2, & \omega^4, & \dots & \omega^{2n} \\ 1, & \omega^3, & \omega^6, & \dots & \omega^{3n} \\ \dots & \dots & \dots & \dots & \dots \\ 1, & \omega^n, & \omega^{2n}, & \dots & \omega^{n^2} \end{vmatrix}$$

(which does not vanish, being equal to the product of the differences of $1, \omega, \omega^2, \dots \omega^n$), where ω denotes a primitive root of the equation $x^{n+1}-1=0$.

Then $$\Delta\Delta' = \Delta \prod_{r=0}^{r=n}(a_0 + a_1 x\omega^r + a_2 x^2\omega^{2r} + \dots a_n x^n \omega^{nr}).$$

Hence if $\lambda_1, \lambda_2, \dots \lambda_n$ be the roots of $a_0 + a_1 x + \dots a_n x^n = 0$,

$$\begin{aligned} \Delta &= \prod_{r=0}^{r=n} a_n \prod_{s=1}^{s=n}(\omega^r x - \lambda_s) \\ &= a_n^{n+1}(-x)^{n(n+1)} \prod_{s=1}^{s=n} \prod_{r=0}^{r=n}\left(\frac{\lambda_s}{x} - \omega^r\right) \\ &= a_n^{n+1} x^{n(n+1)} \prod_{s=1}^{s=n}\left(\frac{\lambda_s^{n+1}}{x^{n+1}} - 1\right) \\ &= a_n^{n+1} \prod_{s=1}^{s=n}\left(\lambda_s^{n+1} - x^{n+1}\right), \end{aligned}$$

a polynomial of order n in x^{n+1}.

Note. This problem is a statement in a slightly altered form of a property noticed by A. R. Forsyth, *Messenger of Mathematics*, Vol. XIV. (1885), pp. 40-56. See also T. Muir in the same volume, and *Quarterly Journal*, Vol. XVIII. (1882), pp. 166-177.

403. [K. 4.] *Construct geometrically a right-angled triangle having given the base b and the rectangle contained by the hypotenuse and the perpendicular.*

ISAAC H. TURRELL.

Solution by W. F. BEARD; H. G. BELL; A. E. WILLIAMS; C. E. YOUNGMAN.

On AB the given base describe rectangle $ABHK$ equal to given rectangle; let F be mid-point of AB; with centre F and radius FH or FK cut AB produced at D. The semicircle on AD will cut BH at C the required vertex. Proof by Eucl. II. 6 and similar triangles. (See the *Gazette*, No. 4, pp. 32, 34.)

405. [K. 12. b. α.] *Draw three circles mutually tangent, which shall also touch a given straight line at given points.* C. E. YOUNGMAN.

Solution by W. F. BEARD; J. C. M. GARNETT.

Let A, B, C be the given points, D the point on AB such that $AD \cdot AC = AB^2$. Describe a $\odot$ C_2 touching AB at B and having its diameter $BB' = BD$. Let AB', CB' cut this $\odot$ again in R, S. Then a $\odot$ C_1 passing through A and touching C_2 at R will touch AB at A (for R is a centre of similitude of C_1, C_2). Similarly a $\odot$ C_3 may be described touching C_2 at S and AB at C. Invert with centre A and radius AB: C_2 inverts into itself; C_1 inverts into C_1' the tangent to C_2 at B'; C_3 into a $\odot$ C_3' touching AB at D and touching also C_2; but DB = diameter of C_2; $\therefore$ C_3' is equal to C_2, and therefore touches C_1'. Hence C_1, C_3 touch and the $\odot$s C_1, C_2, C_3 are the required $\odot$s. [It will be seen that AS, CR meet C_2 again at the extremities of the diameter $\parallel AC$.]

The solution might be obtained by inverting from O the harmonic conjugate of B with respect to A, C, so that A, C, and B become C, A, and B', the mid-point of AC.

407. [L¹. 4. a. a.] *S, H are the foci of an ellipse, T an external point; SM and HN perpendicular respectively to ST and HT meet the adjacent tangents from T to the ellipse in M and N; SN and HM intersect in L: shew that the perpendicular from T on SH passes through L.* (C.)

Solution by W. F. Beard; R. F. Davis.

Draw ME, NF, TR perpendicular to the axis; and let HM meet TR in O. $\angle RTS=\theta$, $RTH=\phi$, $STM=HTN=\alpha$.

Then $SE=SM\cos\theta=ST\tan\alpha\cos\theta=TR\tan\alpha=HF$ by symmetry, and $SF=HE$.

Again $OR.HE=ME.HR=SM\sin\theta.HR=SR.HR\tan\alpha$.

Since OR=symmetrical value, therefore SN passes through O.

Solution by C. E. Youngman.

Suppose STH to remain fixed while the ellipse varies. Then always $\angle STM=HTN$; hence M and N mark homographic ranges; $\therefore$ L moves on a straight line. This line goes through T (for when M is on TH, N is on TS); and through the orthocentre of STH (M and N both at infinity); $\therefore$ it is perpendicular to SH.

408. [L¹. 14. a.] *If the intersections of opposite sides of a circum-quadrilateral of an ellipse lie on a coaxal and similar ellipse, shew that one pair of opposite vertices lie on another coaxal and similar ellipse.* (C.)

Solution by W. F. Beard.

Project orthogonally so that the two given coaxal and similar ellipses become concentric ⊙s. If $ABCD$ be a quadrilateral circumscribing a ⊙ centre O such that, E, F being the intersections of AB with CD and BC with DA, $OE=OF$; then the triangles BAF, BCE will have the angles at F, E equal, the angle at B common, and the sides BF (sum of tangents from B and F), BE (sum of tangents from B and E) equal: hence $BA=BC$; $GA=HC$; and $OA=OC$.

411. [A. 3. k.] *Shew that the roots of*

$$x^3-\frac{p^2x}{3}-\frac{2p^3}{27}-q^2=0$$

differ by a constant quantity from the squares of the roots of

$$x^3+px+q=0.$$ E. Budden.

Solution by R. F. Davis; J. F. Hudson; *and many others.*

If $x^3+px+q=0$, and $y=x^2+e$, then

$$q^2=x^2(x^2+p)^2=(y-e)(y-e+p)^2,$$

which reduces to the given form if $3e=2p$.

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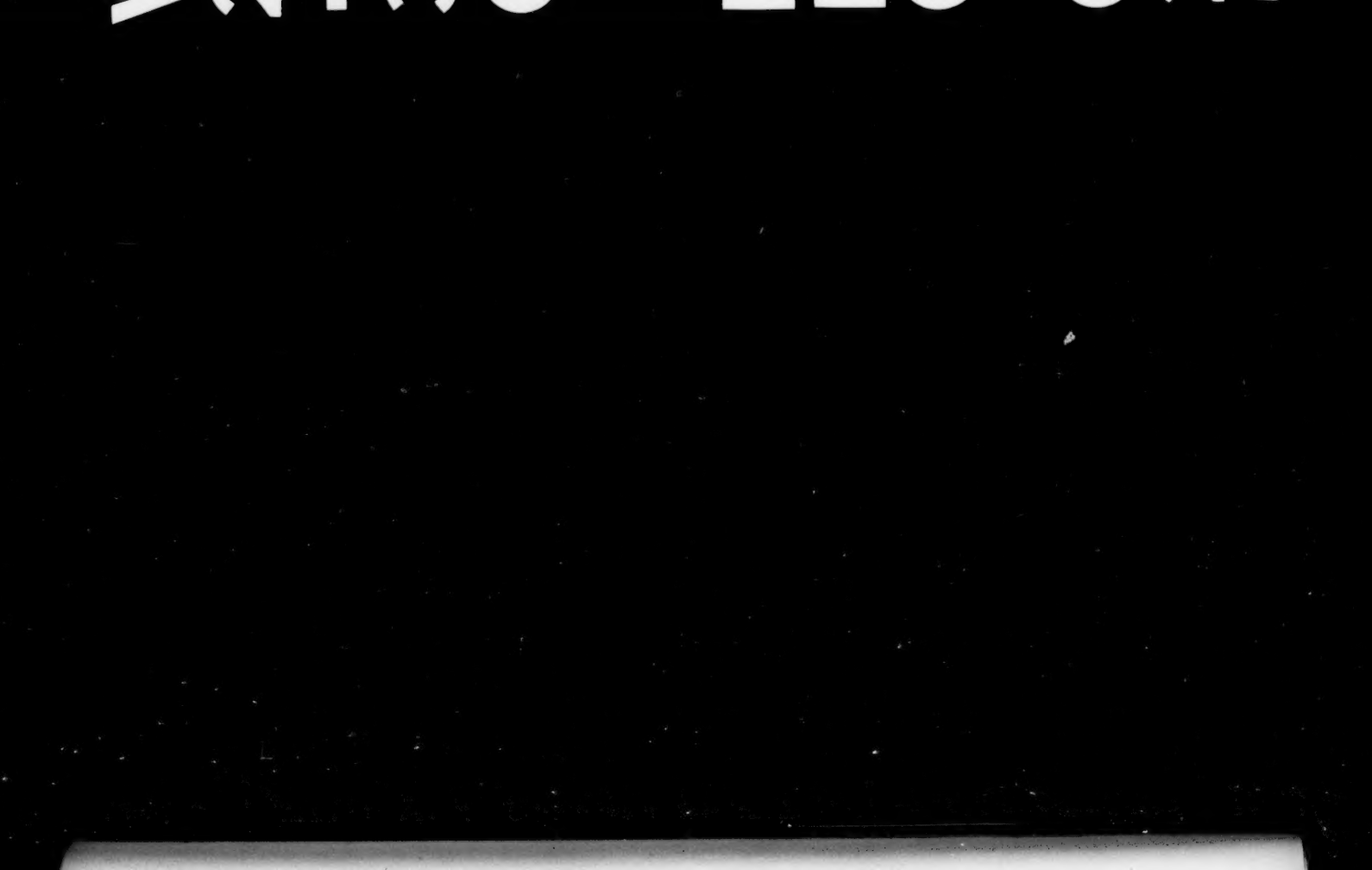
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